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BOUNDARY-VALUE APPROACH TO NUCLEAR EFFECTS IN MUON-CATALYZED D-T FUSION

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ABSTRACT

The use of boundary-matching techniques, as contained in R-matrix theory, to describe multichannel nuclear reactions is discussed. After giving a brief summary of the application of such techniques to the nuclear reactions in the ^{5}He system, we discuss a simple extension of the description to include muons, which was used to calculate nuclear effects on the L=0 eigenvalues of the dtμ molucule in various adiabatic approximations. Next, the form of the outgoing nαμ wavefunction is discussed, resulting in a new formuiation of the amplitudes used to calculate the α–μ sticking fraction. Possible methods of solving for these amplitudes using the boundary-value approach are suggested, and some deficiencies of the "standard" expression for the sticking amplitudes are pointed out.

INTRODUCTION

It is, perhaps, ironic that the fundamental process underlying μ–catalyzed d-t fusion, the nuclear reaction T(d,n)^{4}He, has been one of the last elements added to the descrip'ion of fusion out of the dtμ molecule. In a sense, this is justified by the fact that, although nuclear effects strongly perturb the molecular wavefunction at small d-t separations, the healing distance is short enough that their effects on all the observables of the system considered up until now have been small. In this paper, we will describe how nuclear effects were imposed on the nuclear eigenvalues of the dtμ molecule, and how they could be imposed on a quantity of vital interest in the muon-catalyzed fusion cycle, the α–μ sticking fraction, using an R-matrix description of the nuclear reactions in the ^{5}He system.

We begin with a brief resume of the R-matrix relations and definitions used to characterize the nuclear reactions in the ^{5}He system, followed by a summary of results from an extensive analysis of the experimental data. Next, it is shown how to generalize the R-matrix approach to include the muon, in the approximation that the nuclear and muonic degrees of freedom are separable. We then recapitulate the results of applying the theory, in the reduced R-matrix form, to study nuclear effects on the eigenvalues of the S-wave molecular states, the wavefunctions of which were calculated with a series of increasingly more accurate adiabatic approximations. Finally, we give a new prescription for defining sticking amplitudes, and indicate how they could be obtained numerically from our formalism. Some problems with the "standard" expressions used for the sticking amplitudes are also pointed out.

R-MATRIX DESCRIPTION OF THE NUCLEAR ^{5}HE SYSTEM

R-matrix theory[1] is based on the idea that a many-body nuclear system displays simpler "channel" degrees of freedom whenever the radial separation r_c of two subgroups of the system is increased beyond the channel radius, a_c. In this "channel" region, the subgroups are assumed to be bound in their ground states, with at most (point) Coulomb forces acting between them. The channel radii (a_c) define a channel

surface which encloses the "nuclear" region. On this surface are defined the channel surface states of total angular momentum (J) and parity,

$$(r_c|c) = \left(\frac{1}{2M_c a_c}\right)^{1/2} \frac{\delta(r_c-a_c)}{r_c}\left[\left(\phi_1 \otimes \phi_2\right)_{sv} \otimes i^{\ell} Y_{\ell m}(\hat{r}_c)\right]_{JM} , \qquad (1)$$

in which ϕ_1 and ϕ_2 are the spin-dependent bound-state wavefunctions of the channel, M_c is the channel reduced mass, and $Y_{\ell m}$ is the spherical harmonic describing the orbital motion. The R matrix is then defined as the channel-surface projections of a resolvent (Green's function) operator,

$$R^N_{c'c} = (c'|(H_N + \mathcal{L}_B - E_N)^{-1}|c) = \sum_{\lambda} \frac{\gamma_{c\lambda}\gamma_{c\lambda}}{E_\lambda - E_N} , \qquad (2)$$

in which the boundary operator

$$\mathcal{L}_B = \sum_c |c)(c|(\frac{\partial}{\partial r_c} r_c - B_c) \qquad (3)$$

has been added to the nuclear hamiltonian H_N for total energy E_N in order to make it hermitian in the nuclear region. This allows the spectral expansion made in the last step of Eq. (2) to be defined in terms of the solutions of

$$(H_N + \mathcal{L}_B - E_\lambda)|\lambda) = 0 , \qquad (4)$$

with $\gamma_{c\lambda} = (c|\lambda)$. We use the reduced-width amplitudes $\gamma_{c\lambda}$ and eigenenergies E_λ for boundary conditions B_c at channel radii a_c as a parametric expansion of the R matrix, and determine their values by fitting experimental data.

The formal solution of Schrödinger's equation inside the nuclear region,

$$\psi_N = (H_N + \mathcal{L}_B - E_N)^{-1}\mathcal{L}_B\psi_N , \qquad (5)$$

implies the fundamental R-matrix relation,

$$(c'|\psi_N) = \sum_c R^N_{c'c}(c|(\frac{\partial}{\partial r_c} r_c - B_c)|\psi_N) , \qquad (6)$$

which is a matching condition that can be used to determine quantities of interest in any scattering or bound-state problem. In the case of n coupled channels, the fundamental R-matrix relation holds for each of the n linearly independent solutions ψ^j_N so that in terms of the matrices

$$U_{ij} = (c_i|\psi^j_N) \quad \text{and} \quad U'_{ij} = (c_i|\frac{\partial}{\partial r_i} r_i|\psi^j_N) , \qquad (7)$$

it becomes the matrix relation

$$U = R^N(U' - BU) \qquad \Rightarrow \qquad R^N = (U'U^{-1} - B)^{-1} . \qquad (8)$$

Thus, the R matrix is essentially the reciprocal of a logarithmic derivative matrix, UU^{-1}, for the coupled radial solutions. Any particular solution of the coupled problem can be constructed from a linear combination of the columns of the matrix U.

If one has a multichannel problem in which only a subset of the channels is of explicit interest, the partitioned matrix technique of the reduced R matrix[2] is often useful. This method consists of writing the fundamental R-matrix relation, Eqs. (6) or (8), in partitioned matrix (P,Q, where P+Q=1) form, and assuming that in partition Q,

$$U'_Q = L_Q U_Q \qquad (L_Q = \mathcal{O}'_Q \mathcal{O}_Q^{-1}) \quad , \tag{9}$$

the logarithmic derivatives L_Q are those for purely outgoing-wave solutions $\mathcal{O}_Q$. Then the R-matrix relation for the radial solutions in the P partition can be written as

$$U_P = \{R_{PP} + R_{PQ}(L_Q - B_Q)[1 - R_{QQ}(L_Q - B_Q)]^{-1} R_{QP}\}(U'_P - B_P U_P) \, , \tag{10}$$

which means that the motion in channel group P is described by the reduced R matrix

$$\tilde{R}_{PP} = R_{PP} + R_{PQ}(L_Q - B_Q)[1 - R_{QQ}(L_Q - B_Q)]^{-1} R_{QP} \, . \tag{11}$$

With these preliminary ideas established, we can proceed to a brief description of the application to reactions in the ^{5}He system.[3] A summary of the channel configuration, data included, and χ^2 values for each of the reactions is given in Table I. The important points to note are that: (1) the energy range of the analysis is rather broad, extending to energies well above and below the d-t threshold; (2) many different types of observables (cross sections, polarizations, etc.) are included in the analysis for each reaction, allowing the R-matrix parameters to be determined accurately and unambiguously; (3) the overall χ^2 per degree of freedom for the fit (1.55), while not ideal, indicates in our experience quite a good representation of the measurements for a multichannel system.

Table I Summary of the ^{5}He system R-matrix analysis

Channel	lmax	ac.(fm)
d-t	3	5.1
n-^{4}He	4	3.0
n-^{4}He*	1	5.0

Reaction	Energy Range	# Observable Types	# Data Points	χ^2
T(d,d)T	E_d=0-8 MeV	6	695	1147
T(d,n)^{4}He	E_d=0-8 MeV	13	1020	1423
T(d,n)^{4}He*	E_d=4.8-8 MeV	1	10	17
^{4}He(n,n)^{4}He	E_n=0-28 MeV	2	795	1160
	Totals:	22	2520	3767

parameters = 97 $\Rightarrow$ χ^2 per degree of freedom = 1.55

Examples of the quality of the fits to the integrated cross sections are given in Fig. 1. The top part of the figure shows the fit to measurements of the integrated reaction [T(d,n)^{4}He] cross section, with a comparison at the right side, expressed as a ratio, to the precise new data of Brown and Jarmie[4] over the region of the low-energy resonance. The lower part of the figure shows similar comparisons with the measurements of the n-α total cross section. Again, the right side shows a detail of the fit over the resonance, in this case, containing the recent total cross-section data of Haesner et al.[5] One sees that the R-matrix representation of these data is quite good.

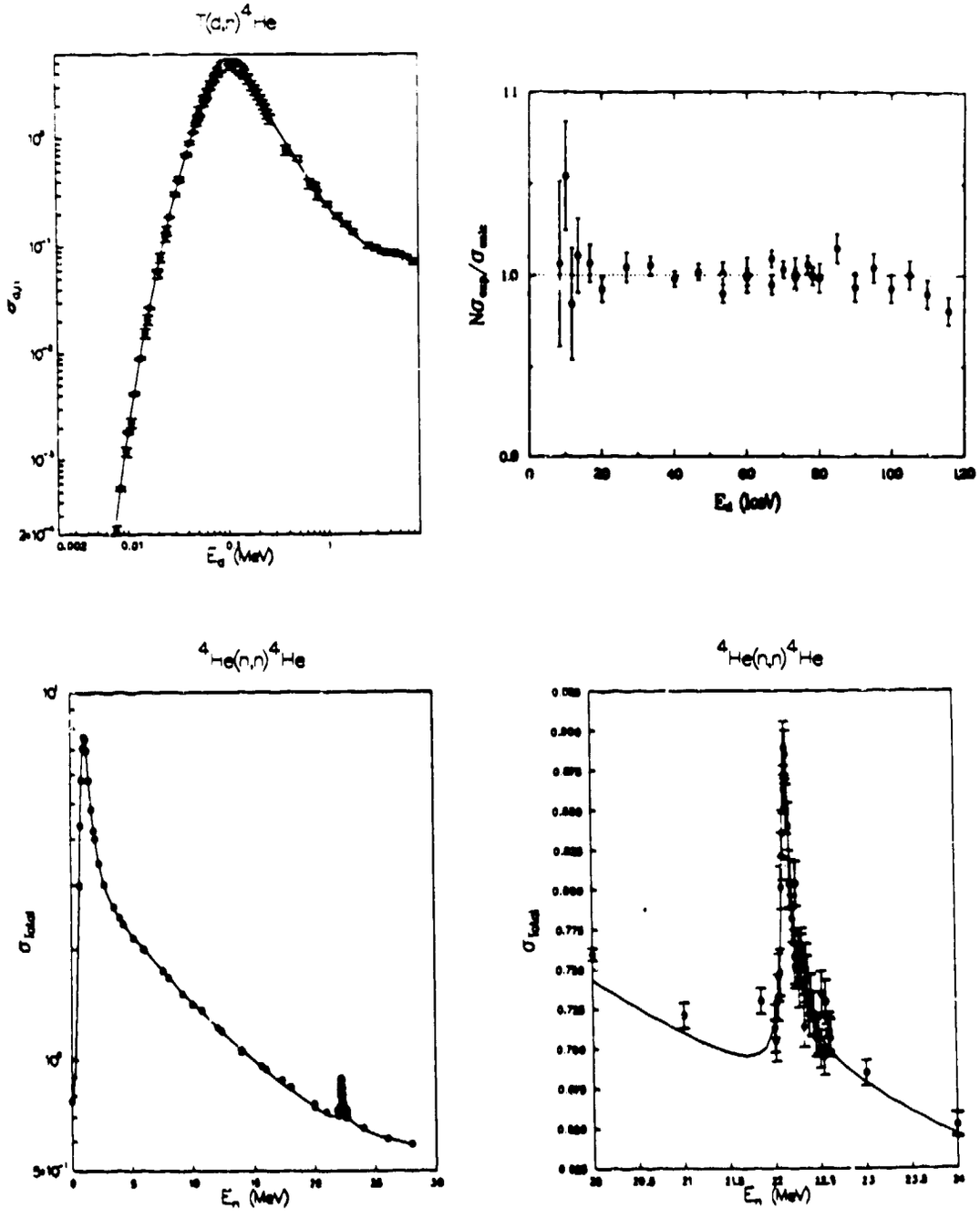

Fig. 1. Fits to measurements of integrated cross sections for the ^{5}He reactions.

At the low relative d-t energies that are of interest in cold fusion, the $T(d,n)^4He$ reaction is almost completely dominated by the resonant $J^\pi=3/2^+$ transition. The R-matrix parameters for this matrix are given in Table II. The first level is mainly associated with the low-energy resonance, the next two with higher-energy ^2D and ^4D resonances in the dt channel, and the fourth serves primarily as a background term, especially in the $n\alpha$ channel. Although the R matrix is clearly the relevant quantity in the muon-catalyzed fusion problem, it is of interest to mention the S-matrix pole structure[6] that results from matching to outgoing waves in the nuclear ^5He system. In addition to a "conventional" pole on the unphysical sheet (Sheet III) closest to the physical sheet of the two-channel Riemann energy surface above the d-t threshold, there is a "shadow" pole on Sheet II (the sheet on which the d-t momentum has a positive imaginary part and the n-α momentum has a negative imaginary part). The effects of the two poles are separately visible in the reaction and total cross sections, as is shown in Ref. 6. The presence of the shadow pole on Sheet II implies that the $J^\pi=3/2^+$ resonance in ^5He would occur only in the $n\alpha$ channel in the absence of coupling between the dt and $n\alpha$ channels, contradicting the usual picture that it is essentially a dt resonance.

Table II R-matrix parameters for the $J^\pi = 3/2^+$ states of ^5He. Channel labels (c) are in spectroscopic notation. Eigenenergies E_λ are center-of-mass values in MeV relative to the d-t threshold; reduced-width amplitudes $\gamma_{c\lambda}$ are also center-of-mass in units $MeV^{1/2}$.

c(J=3/2)	a_c(fm)	B_c		λ=1	λ=2	λ=3	λ=4
			E_λ	0.0837559	6.4713043	13.7357067	47.475246
$^4S(d,)$	5.1	-0.37		1.1760678	0.0693397	-0.4955438	1.1052421
$^4D(dt)$	5.1	-2.00		0.1688724	-0.2729805	1.9910681	1.9847048
$^2D(dt)$	5.1	-2.00	$\gamma_{c\lambda}$	-0.0484797	0.8862475	0.0958513	0.2422464
$^2D(n\alpha)$	3.0	-0.59		0.3763218	-0.1562737	0.9994494	-3.8556539

R-MATRIX DESCRIPTION OF THE $^5HE+\mu$ SYSTEM

The advantage of the R-matrix approach is that the addition of the muon to the system can be treated differently in the nuclear and channel regions. In the nuclear region, where the nucleons are in relatively close proximity, the total hamiltonian H is approximately separable,

$$H \approx H_N + H_\mu \quad , \tag{12}$$

with H_N the nuclear hamiltonian as before, and H_μ the hamiltonian for a muon moving about a ^5He core. The wavefunction in the nuclear region is therefore, in general, a sum of separable terms corresponding to the separation in H. However, we will in the subsequent discussion keep only the first term of this sum,

$$\Psi \approx \psi_N \phi_\mu^0 \quad , \tag{13}$$

in which ϕ_μ^0 is the ground-state μ-^{5}He wavefunction corresponding to the ground-state energy E_μ^0. This approximation joins smoothly with the adiabatic approximations to be made in the channel region, as is depicted schematically in Fig. 2. The R matrix (including the muon) is in this approximation

$$R_{c'c} = (c'|(H_N + \mathcal{L}_B + H_\mu - E)^{-1}|c) = |\phi_\mu^0\rangle R_{c'c}^N(E_N)\langle\phi_\mu^0| \quad , \tag{14}$$

with R^N given by Eq. (2) in terms of the parameters of Table II, and $E_N = E - E_\mu^0$.

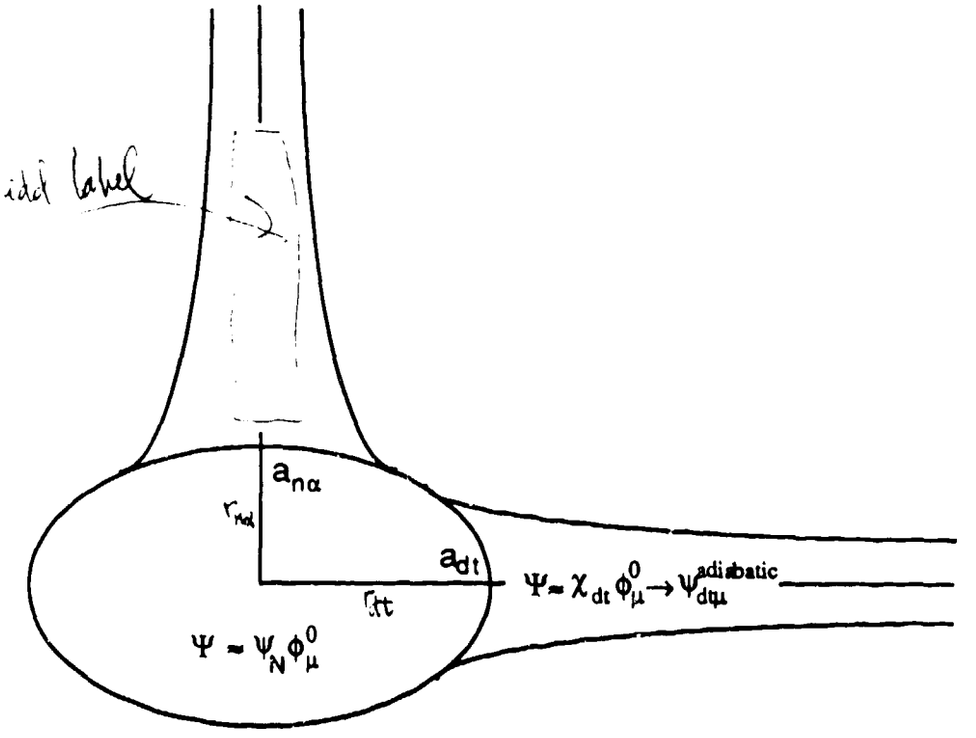

Fig. 2. Schematic of the single muon state (adiabatic) R-matrix approximation

In the channel region, the Coulomb attraction of the muon to the separated nuclear ions has a significant effect, especially in the dt channel, where it causes bound molecular dtμ states to occur. Coulomb binding of muons to alpha particles in the nα channel is also quite important, since this creates the "sticked" muons that are lost from the catalysis cycle. Thus, as the radial distance between the ions grows larger than the channel radius, the separable approximation made in the nuclear region breaks down, and one is faced with the difficult task (at least for dtμ) of solving a three-body Coulomb problem. Exact (non-adiabatic) calculations of bound-state dtμ wavefunctions have been the subject of extensive research over the past decade.[7] However, a complication introduced by the nuclear effects is that, since the channel

region excludes the origin, the wavefunctions in this region contain irregular functions in addition to the regular functions present in the bound states. In order to avoid having to deal with this kind of complexity in a fully non-adiabatic calculation, we decided to test the R-matrix approach using a sequence of increasingly more accurate adiabatic calculations for the dtμ wavefunction in the channel region.

The natural starting point was to consider nuclear effects on the eigenvalues (bound-state energies) E_b of the dtμ molecule in this approximation,[8] which allows the relative d-t wavefunction to be obtained by solving a second-order differential equation containing the muon attraction in an effective d-t potential. The solution of this equation that matches at the channel surface to values derived from the nuclear R matrix and tends asymptotically to a purely outgoing wave is a generalization of the bound-state eigenfunction sometimes called the Siegert state, which occurs for a complex eigenenergy $E_0 = E_r - \frac{1}{2}i\Gamma$. Thus, the shift $\Delta E = E_r - E_b$ and the width Γ (which is proportional to the fusion rate λ_f) are direct measures of the nuclear effect on the bound-state eigenvalues.

The matching at the channel surface is most conveniently accomplished by using the reduced R matrix described in the paragraph containing Eqs. (9)-(11). In this case, we put the $^4S(dt)$ channel (d) of Table II in partition P and the three remaining channels (q=1,2,3) in partition Q, in order to define the single reduced R-matrix element

$$\tilde{R}_{dt\mu} = |\phi_\mu^0\rangle \tilde{R}_{dt}^N \langle\phi_\mu^0| \quad , \tag{15}$$

in which

$$\tilde{R}_{dt}^N = R_{dd}^N + R_{dq}^N(L_q - B_q)[1 - R_{qq}^N(L_q - B_q)]^{-1}R_{qd}^N \quad . \tag{16}$$

Since the adiabatic solutions in the dtμ channel region near $r_{dt} = a_{dt}$ have the form

$$\Psi_{dt\mu}^{adiabatic} = \chi_{dt}(r_{dt})\phi_\mu(r_\mu)$$

in which $\phi_\mu \sim \phi_\mu^0$, we can project the fundamental R-matrix relation, Eq. (6), on $\langle\phi_\mu^0|$ to obtain

$$(dt|\chi_{dt}\rangle = \tilde{R}_{dt}^N (dt|(\frac{\partial}{\partial r_{dt}} r_{dt} - B_{dt})|\chi_{dt}\rangle \tag{17}$$

as a matching condition in the dt channel at $r_{dt} = a_{dt} = 5.1$ fm.

The purely outgoing-wave solutions for $r_{dt} > a_{dt}$ were calculated using three successively more realistic adiabatic approximations: the Born-Oppenheimer (BO), standard adiabatic (SA), and improved adiabatic (IA) approximations. The improvements on the familiar BO approximation involved adding diagonal pieces of the hamiltonian to the definiton of the adiabatic potentials and using angular-dependent muonic wavefunctions, the details of which are contained in Ref. 8. In each case, the complex energy E_0 at which the matching equation (17) is satisfied was determined by iteration. Then, by comparing with the bound-state eigenvalues obtained by matching to regular boundary conditions at the origin, the shifts and widths arising from matching to nuclear boundary conditions at the channel radius were found.

The results for the lowest vibrational-rotational (v,L) states are given in Table III for L=0 and v=0,1. The numbers suggest that the SA and IA approximations are converging to something like the correct results, as was also indicated by a comparison of the adiabatic bound-state energies to the non-adiabatic value, and of the approximate μ-^{5}He energies to E_μ^0. The agreement with the earlier calculations of Bogdanova et al.[9] is quite stisfactory, considering that these calculations were done using an optical-

potential representation of the nuclear interactions, and different approximations were made in solving for the dtμ wavefunction and the complex eigenvalue E_0. Also shown are the more recent results of Karnikov and Mur,[10] who used the single-level R-matrix representation of the reaction cross section given by Brown and Jarmie.[4]

TABLE III Energy shifts and resonance widths for the (v=0, L=0) and (v=1, L=0) vibrational-rotational states due to nuclear boundary conditions. Energies are in units of 10^{-4} eV.

	$\Delta E^{(v=0,L=0)}$	$\Gamma^{(v=0,L=0)}$	$\Delta E^{(v=1,L=0)}$	$\Gamma^{(v=1,L=0)}$
BO	-9.87	10.35	-7.63	8.03
SA	-8.15	8.53	-7.08	7.44
IA	-8.12	8.50	-6.75	7.10
Ref. 9	-9.6	8.2	-8.0	6.8
Ref. 10	-12.5	9.0	-10.4	7.4

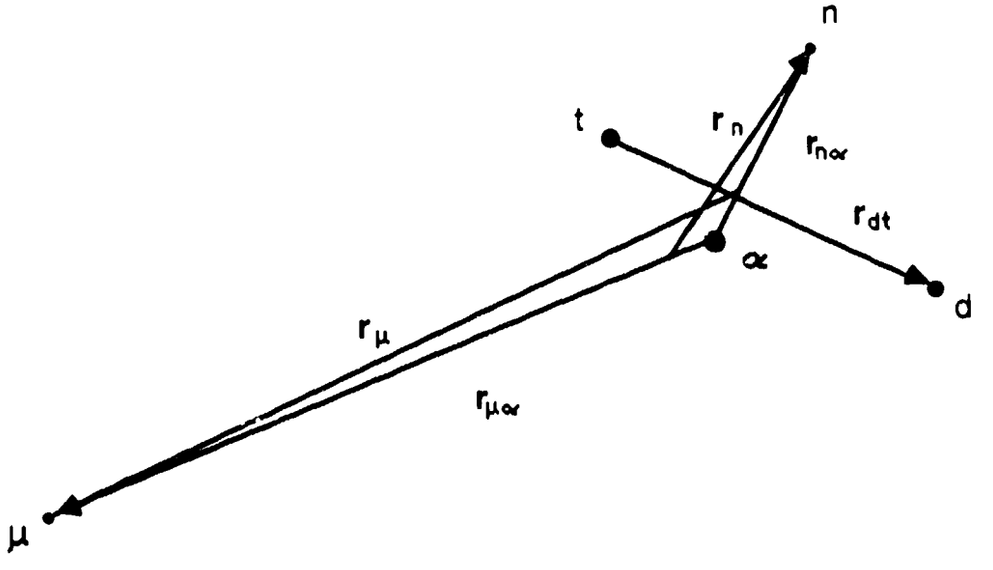

Fig. 3. Coordinates for describing the channel regions of ^{5}He+μ.

WAVEFUNCTION IN THE nαμ CHANNEL REGION AND μ-α STICKING

The coordinate labeling used to describe the channel regions of the system is given in Fig. 3. At the nα channel surface, $r_{n\alpha}=a_{n\alpha}=3$ fm, the outgoing wave can be expressed in the separable approximation as

$$\Psi_{n\alpha\mu}(\mathbf{r}_{n\alpha},\mathbf{r}_{\mu}) = \beta h_2^+(k_{n\alpha}r_{n\alpha})Y_{20}(\hat{\mathbf{r}}_{n\alpha})\,\phi_{\mu}^{0}(\mathbf{r}_{\mu}) \quad , \tag{18}$$

in which β is a constant and h_2^+ is the outgoing Hankel function for $\ell=2$. For $r_{n\alpha}>a_{n\alpha}$, however, since the hamiltonian is exactly separable in the variables $(\mathbf{r}_n, \mathbf{r}_{\mu\alpha})$, the wavefunction can be written as

$$\Psi_{n\alpha\mu}(E) = \int dE_n \int d\hat{\mathbf{k}}_n \hat{\chi}_n(\mathbf{k}_n,\mathbf{r}_n) \int d\rho(E_{\mu\alpha})\phi_{\mu\alpha}(E_{\mu\alpha},\mathbf{r}_{\mu\alpha})a_E(\hat{\mathbf{k}}_n,E_{\mu\alpha}) \times \delta(E_n+E_{\mu\alpha}-E)$$

$$= \int d\hat{\mathbf{k}}_n \int d\rho(E_{\mu\alpha}) a_E(\hat{\mathbf{k}}_n,E_{\mu\alpha})\hat{\chi}_n(\hat{\mathbf{k}}_n,\mathbf{r}_n)\phi_{\mu\alpha}(E_{\mu\alpha},\mathbf{r}_{\mu\alpha})|_{E_n=E-E_{\mu\alpha}} \tag{19}$$

in terms of the energy and momentum variables conjugate to $\mathbf{r}_n$ and $\mathbf{r}_{\mu\alpha}$. In Eq. (19), the plane-wave states $\hat{\chi}_n$ and μ-α states $\phi_{\mu\alpha}$ are taken to be energy-normalized, as is $\Psi_{n\alpha\mu}$, so that the energy-conserving delta function in the first line results naturally from the normalization conventions. The energy normalization of $\Psi_{n\alpha\mu}$ also implies the relation

$$\int d\hat{\mathbf{k}}_n \int d\rho(E_{\mu\alpha}) |a_E(\hat{\mathbf{k}}_n,E_{\mu\alpha})|^2 = 1 \tag{20}$$

for the squared modulus of the expansion amplitudes a_E integrated over all neutron directions $\hat{\mathbf{k}}_n$ and over the density $\rho(E_{\mu\alpha})$ of all bound and continuum μ-α states. This means that the sum over the bound states,

$$\omega_s^0 = \sum_b \int d\hat{\mathbf{k}}_n |a_E(\hat{\mathbf{k}}_n,E_{\mu\alpha}^b)|^2 \tag{21}$$

is the sticking fraction, and $a_E(\hat{\mathbf{k}}_n, E_{\mu\alpha}^b)$ is the sticking amplitude for direction $\hat{\mathbf{k}}_n$.

One could invert the expansion for $\Psi_{n\alpha\mu}$ in Eq. (19) and find the sticking amplitude from the integral

$$a_E(\hat{\mathbf{k}}_n,E_{\mu\alpha}^b) = \int d\mathbf{r}_n \hat{\chi}_n^*(\hat{\mathbf{k}}_n,\mathbf{r}_n) \int d\mathbf{r}_{\mu\alpha}\phi_{\mu\alpha}^b(E_{\mu\alpha}^b,\mathbf{r}_{\mu\alpha})\Psi_{n\alpha\mu}(E) \quad , \tag{22}$$

but this requires knowing $\Psi_{n\alpha\mu}(E)$ for all $\mathbf{r}_n$ and $\mathbf{r}_{\mu\alpha}$ (including the nuclear region). Alternatively, one could project the expansion for $\Psi_{n\alpha\mu}$ onto a complete set of channel-surface functions

$$(\mathbf{r}_{n\alpha},\mathbf{r}_{\mu}|n\alpha\mu) = \frac{\delta(r_{n\alpha}-a_{n\alpha})}{r_{n\alpha}} Y_{\ell 0}(\hat{\mathbf{r}}_{n\alpha})\phi_{\mu}^{j}(\mathbf{r}_{\mu}) \quad ,$$

in which the index j ranges over all bound and continuum μ-^{5}He states, and obtain an infinite set of coupled equations for the sticking amplitudes (after a partial-wave expansion in $\hat{\mathbf{k}}_n$). If the coupling terms $(n\alpha\mu|\chi_n\,\phi_{\mu\alpha})$ on the right side of the equation

vanish for high-lying states ϕ^{j}_{μ} and $\phi_{\mu\alpha}$, then the truncated system of equations can be solved by matrix inversion, knowing the left-hand side matrix elements,

$$(n\alpha\mu|\Psi_{n\alpha\mu}\rangle = \beta h^{+}_{\ell}(k_{n\alpha}a_{n\alpha})\delta_{\ell 2}\delta_{j0} \quad ,$$

vanish if the n-α angular momentum differs from $\ell = 2$ or if the μ-^{5}He wavefunction is not that for the ground state.

The sticking amplitudes and sticking fraction defined in Eqs. (19) and (21) are, in principle, quite different from the usual expressions used[11], which are based on the impulse-approximation transition matrix element

$$a_{n\ell} = \langle \phi^{n\ell}_{\mu\alpha}(\mathbf{r}_{\mu}) e^{i\mathbf{q}\cdot\mathbf{r}_{\mu}} | \Psi_{dt\mu}(\mathbf{r}_{\mu}, \mathbf{r}_{dt}=\mathbf{0}) \rangle \quad , \tag{23}$$

for the (n,ℓ) μ-α bound state and muon momentum $\mathbf{q}$. However, one can obtain from our framework expressions similar to those usually used by making the plane-wave approximation for $\Psi_{n\alpha\mu}$,

$$\Psi_{n\alpha\mu} \propto e^{i\mathbf{k}_{n\alpha}\cdot\mathbf{r}_{n\alpha}}\phi^{0}_{\mu}(\mathbf{r}_{\mu\alpha}) \quad .$$

In this case,

$$\langle \hat{\chi}_{n}(\mathbf{k}_{n})\phi^{b}_{\mu\alpha}|\Psi_{n\alpha\mu}\rangle = \delta(\mathbf{k}_{n\alpha}-\mathbf{k}_{n})\langle \phi^{b}_{\mu\alpha}|e^{i\gamma_{\mu}\mathbf{k}_{n\alpha}\cdot\mathbf{r}_{\mu\alpha}}|\phi^{0}_{\mu}\rangle \quad ,$$

and the leading delta function is a poor approximation to the energy-conserving $\delta(E_n+E_{\mu\alpha}-E)$ expected, with $E = E_{n\alpha}+E^{0}_{\mu}$. The sticking amplitude is then

$$\begin{aligned} a_{E}(\hat{\mathbf{k}}_{n}, E^{b}_{\mu\alpha}) &= \int dE_{n}\,\delta(\mathbf{k}_{n\alpha}-\mathbf{k}_{n})\langle \phi^{b}_{\mu\alpha}|e^{i\gamma_{\mu}\mathbf{k}_{n\alpha}\cdot\mathbf{r}_{\mu\alpha}}|\phi^{0}_{\mu}\rangle \\ &= \delta(\hat{\mathbf{k}}_{n\alpha}-\hat{\mathbf{k}}_{n})\langle \phi^{b}_{\mu\alpha}|e^{i\gamma_{\mu}\mathbf{k}_{n\alpha}\cdot\mathbf{r}_{\mu\alpha}}|\phi^{0}_{\mu}\rangle\,|_{E_n=(1-\alpha)E_{n\alpha}} \quad . \end{aligned} \tag{24}$$

In these expressions, the mass ratios $\gamma_i = \frac{m_i}{m_i+m_{\alpha}}$ and $\alpha = \gamma_{\mu}\gamma_n = 0.0055$ are used. The matrix element in Eq. (24) is energy-conserving only for $E_{\mu\alpha} = \alpha E_{n\alpha} + E^{0}_{\mu} = 86$ keV, far above the bound-state energies at which it is evaluated. However, if the delta functions are ignored, the remaining matrix element,

$$a_{E}(E^{b}_{\mu\alpha}) = \langle \phi^{b}_{\mu\alpha}|e^{i\gamma_{\mu}\mathbf{k}_{n\alpha}\cdot\mathbf{r}_{\mu\alpha}}|\phi^{0}_{\mu}\rangle \quad ,$$

is similar to the standard expression for the sticking amplitude.

SUMMARY AND CONCLUSIONS

We have discussed the R-matrix parameterization of the two-body reactions of a multichannel system, and have shown that such a description gives a detailed and accurate representation of the experimental measurements for the ^{5}He system. The "shadow pole" associated with $J^{\pi}=3/2^{+}$ resonance in the nuclear ^{5}He system is an interesting phenomenon that could have consequences for muon-catalyzed fusion, since it occurs on the same sheet as the dtμ molecular ground state.

The R-matrix framework is well-suited also for describing the ^{5}He+μ system, since the division of coordinate space into the nuclear and channel regions allows a relatively simple separable approximation to be made in the nuclear region that is not valid in the channel regions. On the other hand, the three-body wavefunctions in the channel regions can be treated exactly, albeit with some difficulty in the case of dtμ. Our